

UNSTEADY FREE CONVECTION
ON A HORIZONTAL CIRCULAR CYLINDER
IN THE PRESENCE OF HEAT GENERATION AND RADIATION

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For my beloved mother



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ABSTRACT

The unsteady free convective flow about a horizontal circular cylinder in the presence of heat generation and radiation is considered in this thesis. The cylinder is fixed and immersed in a Newtonian fluid, while the temperature at the surface is either heated uniformly or oscillated harmonically about the mean temperature, temperature of the surrounding. In this study, the governing equations are first transformed into a non-dimensional form by using appropriate non-dimensional variables. These equations are then transformed into a system of nonlinear partial differential equations, then were solved numerically by using two types of finite difference methods namely explicit finite difference method and implicit finite difference scheme of Crank Nicolson method. The solutions obtained by those two methods are used to compare the accuracy between them with the previous studies. The presence of heat generation significantly gives effect which increased the local skin-friction coefficient, temperature and velocity distribution but the local rate of heat transfer decreases. While, the presence of radiation increase the local rate of heat transfer, skin-friction coefficient, the temperature and velocity distributions. On the other hand, as the Prandtl number increases, the temperature and velocity distributions are decreases. This study also presents the temperature patterns for the problems when the temperature of the surface is heated and oscillated with respect to time. The comparison between the presence of radiation, heat generation and both of radiation and heat generation on temperature profiles against time are presented. It would be interesting when this study extended by involving the effect of magnetic field on this study and solve the related problems by using other numerical method.

ABSTRAK

Kajian ini berkaitan aliran olakan bebas yang tidak mantap terhadap silinder membulat dan mengufuk dengan kewujudan penjanaan haba dan radiasi. Silinder diletakkan di dalam bendalir Newtonan, manakala suhu pada permukaannya sama ada dipanaskan secara seragam atau berayun secara harmonik terhadap suhu di sekitarnya. Di dalam kajian ini, persamaan yang diperolehi akan dibentuk ke dalam bentuk tak berdimensi dengan menggunakan pembolehubah tak berdimensi yang sesuai. Persamaan menakluk ini kemudiannya dijemakan menjadi satu sistem persamaan pembezaan separa tak linear, dan seterusnya diselesaikan secara berangka menggunakan dua jenis kaedah pembeza terhingga iaitu kaedah beza terhingga tak tersirat dan kaedah beza terhingga yang tersirat jenis Crank Nicolson. Penyelesaian yang diperolehi dari kedua-dua kaedah ini dibandingkan ketepatannya berbanding dengan kajian sebelum ini. Kewujudan penjanaan haba memberi kesan yang ketara iaitu meningkatkan lagi nilai pekali geseran kulit, taburan suhu dan taburan halaju tetapi mengurangkan kadar pemindahan haba. Manakala, kehadiran radiasi pula meningkatkan kadar pemindahan haba, pekali geseran kulit, taburan suhu dan taburan halaju. Sebaliknya, dengan penambahan nombor Prandtl, taburan suhu dan taburan halaju menurun. Kajian ini juga menunjukkan corak taburan suhu apabila suhu permukaan dipanaskan dan berayun terhadap masa. Perbandingan antara kesan radiasi, penjanaan haba dan kedua-dua radiasi dan penjanaan haba pada taburan suhu melawan masa juga diperolehi. Kajian ini menjadi menarik untuk dilanjutkan dengan melibatkan kesan medan magnet dan menyelesaikan masalah-masalah yang berkaitan dengan menggunakan kaedah berangka yang lain .

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NOMENCLATURE

Roman Letters

A	-	Surface area
a	-	Radius of the circular cylinder
C_p	-	Specific heat at constant pressure
C_f	-	Skin-friction coefficient
e	-	Specific internal energy
F_s	-	Surface forces
$F_{s,x}$	-	Body forces on x -direction
$F_{s,y}$	-	Body forces on y -direction
g	-	Acceleration due to gravity
Gr	-	Grashof number
M	-	Rate of mass transfer
Nu	-	Nusselt number
P	-	Pressure
P_d	-	Dynamic pressure
P_h	-	Hydrostatic pressure
Pr	-	Prandtl number
\bar{q}	-	The volumetric of heat generation
q''_x	-	Heat flux for x direction
q''_y	-	Heat flux for y direction
q''_s	-	Heat transfer rate
q''_r	-	Radiation heat flux
Q_0	-	The heat generation constant
R_d	-	Radiation parameter
t^*	-	Time
T^*	-	Fluid temperature

T_{∞}^*	-	Ambient temperature
T_s^*	-	Surface temperature
t	-	Non-dimensional time
T	-	Non-dimensional temperature of fluid
T_{∞}	-	Non-dimensional temperature of the ambient fluid
T_s	-	Non-dimensional temperature of the surface
u, v	-	Velocity components along the x and y directions, respectively
x, y	-	Cartesian coordinates along the surface and normal to it, respectively
U, V	-	Non-dimensional velocity components along the x and y directions, respectively
X, Y	-	Non-dimensional cartesian coordinates along the surface and normal to it, respectively
\mathbf{X}, \mathbf{Y}	-	Body force vector per unit area x and y , respectively

Greek Letters

κ	-	Thermal conductivity
ε	-	Material property
σ	-	Normal stress
σ_{sb}	-	Stefan-Boltzmann constant
σ_s	-	The scattering coefficient
α	-	Effective thermal diffusivity
α_r	-	Absorption coefficient
δ	-	Control area
Δ	-	Difference between two properties
γ	-	Heat generation parameter
β	-	Thermal expansion coefficient
ω	-	Frequency of the oscillation
ρ	-	Fluid density
ρ_{∞}	-	Density of the ambient fluid

Φ	-	Viscous dissipation function
ν	-	Kinematic viscosity
μ	-	Dynamic viscosity
∇^2	-	Laplace operator
τ	-	Shear stress
τ_s	-	Shear stress on the surface

Superscripts

*	-	Dimensional condition
"	-	Derivative

Subscripts

∞	-	Far field condition
s	-	Surface
i, j	-	Integer for x and y direction, respectively



CHAPTER 1

INTRODUCTION

1.1 Introduction

The unsteady free convective flow problems are important in engineering applications. There are many researches on free convection over a circular cylinder but less studies in the presence of heat generation or radiation or both of them. Many of the problems study the effect of heat generation and radiation in uniform surface temperature or steady state condition but not in the case of unsteady flow or non uniform temperature.

Free convection about a circular cylinder can be found in the application of technology where the temperature of the surface are either oscillated or heated. The problems for unsteady free convection over circular cylinder can be solved either analytically or numerically. Most of the previous researchers have studied the oscillatory or unsteady free convection on different types of shapes without the presence of heat generation or radiation.

Besides that, most of the studies employed a numerical solution but on different types of geometrical shape and the temperature of the surface in the case of steady condition. However, until now less amount of works has been done by considering the unsteady free convection over a horizontal circular cylinder with employed analytic solutions. Even the earlier studies were about an unsteady problems, the effect of heat generation and radiation excluded in their study. They were studied on different types of geometrical shapes. However, the study on the effect of heat generation and radiation has been investigated in the case of steady state natural convective flow. The unsteady free convection about a circular cylinder with the presence of heat generation

and radiation is new and has not been studied by any researcher. The governing equations and boundary equations are first transformed into a dimensionless form, and the resulting system of equations are then solved by a numerical method in their study.

Natural convection is interested and given an attention from researchers because it applied in engineering applications. Natural convection is also plays an important role in fluid flows such as the free-convection cooling of electronic components, the spreading of pollutants from smoke stacks, and the human thermal plume. Other than that, in engineering applications, convection is commonly used in the formation of microstructures during the cooling of molten metals, and fluid flows around shrouded heat-dissipation fins, and solar ponds.

The presence of heat generation on free convection can be seen its significance in the contexts of space technology and processes involving high temperature. The effects of heat generation may alter the temperature distribution and, therefore, the particle deposition rate. This may occur in such applications related to nuclear reactor cores, fire and combustion modelling, electronic chips and semiconductor wafers.

1.2 Problem nature

In this thesis, unsteady free convection about an infinite horizontal circular cylinder with the presence of heat generation and radiation will be studied as a model problem. The infinite horizontal circular cylinder is fixed and hence is assumed to be in a two-dimensional problem. The circular cylinder is immersed in Newtonian fluid and the temperature of the surface of the cylinder is either heated uniformly or oscillated harmonically with the presence of heat generation and radiation. The domain of the solution is around the circular cylinder. The dimensionless non-linear systems of partial differential equations subject to the boundary conditions given will be solved numerically. The solutions obtained in the present study is on the effect of heat generation, radiation and Prandtl number on the velocity and temperature distribution, the local of heat transfer and the skin-friction coefficient.

1.3 Objectives

The objectives of this research are to develop the mathematical model and find the numerical solutions for the problems on unsteady free convection around horizontal circular cylinder where the surface temperature is either uniformly heated or oscillated harmonically with the presence of heat generation and radiation. The numerical algorithms are computed by computer programming FORTRAN95 for the following problems:

- (i) Unsteady free convection around horizontal circular cylinder where the surface temperature is heated uniformly with the presence of heat generation.
- (ii) Oscillatory free convection around horizontal circular cylinder with the presence of heat generation.
- (iii) Unsteady free convection around horizontal circular cylinder where the surface temperature is heated uniformly with the presence of radiation.
- (iv) Unsteady free convection around horizontal circular cylinder where the surface temperature is heated uniformly with the presence of heat generation and radiation.

1.4 Scopes of study

The scope of study is limited to problems involving unsteady, two-dimensional free convection around horizontal circular cylinder, immersed in viscous and incompressible fluid with two types of boundary conditions namely heated uniformly and oscillated harmonically surface temperature in the presence of heat generation and radiation. These problems are formulated using the nonsimilar transformation and solved numerically using finite difference method.

1.5 Introduction to heat transfer

Heat transfer described as exchange the thermal energy in the form of heat through a body or between bodies. Heat transfer is energy in transit due to a temperature gradient or difference. When two bodies are at different temperatures, thermal energy transfers from the one with higher temperature to the one with lower temperature. For example in our daily life, we can see heat always moves from hot to cold and cold objects in a warmer room will heat up to room temperature. There are three basic mechanisms or modes of heat transfer: conduction, convection, and radiation as reported in Bejan (1993).

1.5.1 Conduction

Conduction is the transmission of heat through a substance without perceptible motion of the substance itself (Janna (2000)). The heat transfer via conduction made the substance itself does not flow because the heat is transferred internally, by vibrations of atoms and molecules. Electrons can also carry heat so that, metals are generally very good conductors of heat. This is because metals have many free electrons and move randomly thus, the heat was transferred from one part of the metal to another part. The example of conduction heat transfer in our daily life is when we touch a hot object, we will feel the heat on our skin because the heat from the object is transferred through our skin by conduction.

There are two mechanisms in heat transfer via conduction namely, lattice vibration and particle collision. Lattice vibration occurs in the process of heat transfer in solids. In solids, the atoms are bound to each other by a series of bonds. As the solid heated, there is the temperature gradient or difference in the solid made the atom vibrates at the hot side of the solid. Then, those vibrations make the adjacent atoms vibrate, and so on, the vibrations are transmitted through the springs to the cooler side of the solid and reach an equilibrium, where all the atoms are vibrating with the same energy.

In the case of metals which have free movement electrons and not bound to any electrons in the solid, the heat transferred occurs by particle collision. The electrons

in the hot side of the solid move faster than those on the cooler side. As the electrons undergo a series of collisions, the faster electrons give off some of their energy to the slower electrons. Because of these collisions, the slower electrons move faster and heat transferred until it reach an equilibrium, where the electrons are moving at the same average velocity. Conduction through electron collision is more effective than through lattice vibration. Conduction of heat in liquids is the same as for gases which occurs through collisions between freely moving molecules. The mechanism is similarly to the electron collisions in metals.

The law of heat conduction, also known as Fourier's law, states that the time rate of heat transfer through a material is proportional to the negative local temperature gradient and to the area, at right angles to that gradient, through which the heat flows. Therefore, the rate of heat transfer by conduction in x direction is given by Janna (2000) and Bejan (1993) as

$$q_x'' = -\kappa \frac{\partial T^*}{\partial x}, \quad (1.1)$$

where ∂T^* is the temperature difference between the two surfaces separated by a distance ∂y and κ is the thermal conductivity coefficient. The effectiveness by which heat is transferred through a material is measured by the thermal conductivity, κ .

1.5.2 Convection

Convection is about the heat transport effected by the flow of fluids (Bejan (2004)). The process of heat transfer between a surface and a moving fluid from one place to another place when they are in different temperature. The presence of temperature gradient will contributes heat transfer from a surface to a moving fluid due to bulk motion of the fluid. Convection cannot occurs in solids because there is no bulk flow. The term convection is referring to heat transfer with any fluid movement either liquid or gases, but the term advection is more precise for the transfer due to the bulk fluid flow. The process of heat transfer via convection is a heat transfer from a solid to a fluid with requiring diffusion or conduction of heat through the boundary layer and also advection of heat.

The convection heat transfer occurs at different temperature that will develop

a bounding surface or boundary layer. When there is temperature difference between the temperature of wall with the fluid, the interaction of fluid and the surface occurred made a developed region in fluid where the velocity varies from zero at the surface to a finite value which associated with the flow. Those regions are the velocity boundary layer. While, the thermal boundary layer developed at the region of the fluid where the major temperature and concentration changes occur in that region which is very close to the surface. The thermal boundary layer may be smaller than or larger than or the same size as the velocity boundary layer. Through the analysis of convective heat transfer, convection process can be divided into two natural convection and forced convection (Amer Nordin Darus (1995)). The natural convection or free convection is the flow of fluid which depend to density differences caused by temperature differences to transfer the heat. While, the forced convection is the fluid flow caused by and external agency such as a fan or pump or due to atmospheric disturbances, that heat transfer known as forced convection.

Natural convection or free convection occurs due to the differences of temperature which affect the density and thus the relative buoyancy of the fluid (Bejan (1993)). The heavier components that have more density will fall while the lighter components which have less density will rises and lead to bulk fluid movement. Natural convection can only occur in a gravitational field. In increasing temperature will produce the different densities of fluids that were affected by gravity. Natural convection will be less likely or less rapid with more rapid diffusion or more viscous fluid. There are three cases that deal with natural convection, first is about problems of heat exchange between a body and an extensive quiescent ambient fluid or natural convection on external surfaces. Second is an open-cavity problem, such as natural convection in arrays or through cooling slots and the last case deals with natural convection in enclosures, such as in the annulus between cylinders. The first case of natural convection is in the scope of this thesis and will be discussed further in the next section.

Free convection is quite important because of its widespread application, and the convection heat transfer processes is relevance to industrial and environmental problems. We can found applications of free convection in our daily life for example, in household appliances such as electric kettle and radiator. The electric kettle is heated from the bottom. So that, there is convection heat transfer in the water to the air when

the water boiled. The same reason goes to radiator. For a hot radiator, the air around the radiator is heated and thus expands. The convection works by circulating the hot water around the radiator. Car engines are also cooled by convection currents in the water pipes. The heat exchanged will carries the unwanted heat to the radiator by water.

1.5.3 Radiation

Radiation is heat transfer from a distance without making direct contact which that does not rely upon any contact between the heat source (Bejan (1993)). Radiation allows energy to be transferred either to a surface or from a surface by absorption or emission through wave energy or it may be called as electromagnetic waves because the energy travels in a combination of electric and magnetic waves. This energy is released when these waves are absorbed by an object. The radiation with longer wavelengths generally can penetrate through thicker solids.

Radiation can even work through the vacuum of space for examples the radiation is the heat from the sun, the heat is radiated through space to our planet without the aid of fluids or solids. The rate at which this energy is released is proportional to the Kelvin temperature (T) raised to the fourth power.

$$\text{Radiation rate} = \kappa T^4. \quad (1.2)$$

Most of the objects with high temperature, from a cooking standpoint, emit infrared radiation. Hotter objects, such as the sun, emits more energetic radiation including visible and UV. So that, the amount of radiation emitted by an object is given by Mahan (2002) as

$$q_{\text{emitted}} = \varepsilon \sigma_{sb} \cdot A T_s^4, \quad (1.3)$$

where A is the surface area, T_s is the temperature of the body, σ_{sb} is the Stefan-Boltzmann constant and ε is a material property called emissivity. The radiation that is absorbed by the object is called the absorptivity, α_r . Thus, the amount of heat absorbed by the surface is given by

$$q_{\text{absorbed}} = \alpha_r \cdot I, \quad (1.4)$$

where I is the incident radiation which can be determined by the amount of radiation emitted by the object and strikes the surface. The difference between the rates of radiation emitted by the surface and the radiation absorbed is the net radiation heat transfer. If the rate of radiation absorption is greater than the rate of radiation emission, the surface is said to be gaining energy by radiation. Otherwise, the surface is said to be losing energy by radiation.

Most researchers interested on radiation interaction with convection for heat and mass transfer in fluids. This is due to the significant role of thermal radiation in the surface heat transfer when convection heat transfer is small, particularly in free convection problems involving absorbing emitting fluids. Radiation is important in some applications because of the manner in which radiant emission depends on temperature. For example, radiation contributes substantially to energy transfer in combustion chambers, fires and to the energy emission from a nuclear explosion. We can see in our nature, the radiation from the sun is important to the technology for solar-energy utilization. Solar energy transferred through the vacuum of space and the earth's atmosphere is received by a solar collector that converts the solar radiation into internal energy. This study is strong motivations for studying about radiation because it gives humans the benefits in their life.

1.6 Fluid dynamics for natural convection heat transfer

Fluid dynamics is one part of the fluid mechanics that deal in fluid flow. The fluid motion in free convection problem considers the process of heat transfer between a solid and a liquid or gas flow. The processes that involve the dynamic of fluids will be necessary to understand and described in mathematical terms or mathematical models.

The solution for a fluid dynamics problem in free convection involves calculation on the properties of the fluid such as velocity, pressure, density and temperature, as functions of space and time. The fundamental principles of fluid dynamics are a conservation laws including conservation of mass, conservation of linear momentum (also known as Newton's Second Law of Motion) and conservation of energy (also known as First Law of Thermo-dynamics) lead to the governing equations.

The first principles of fluid dynamics are the conservation of mass or the continuity of mass through a flow system. The continuity equation can be described as an equation for a variety of physical phenomena in transport of a conserved quantity. The velocity and temperature distribution in a flow region near a surface are usually interested in convective heat transfer. In free convection problems, the Navier-Stokes equations are the momentum equation for Newtonian fluids. The Navier-Stokes equations arise from applying Newton's second law to fluid motion. Navier-Stokes is a non-linear set of differential equations that describe the flow of a fluid whose stress depends linearly on velocity gradients and pressure.

Free convection process can be classified as internal or external flow. External flow is flows over bodies that immersed in a fluid, (plate, cylinder and sphere) and internal flow is the flow in the tube. The resulting of both flow may be laminar flow or turbulent flow and compressible or in-compressible (Bejan (2004)). A laminar flow defined as the fluid particles move in smooth layers while a turbulent flow is the fluid particles rapidly mix as they move along due to random three-dimensional velocity fluctuations. In-compressible defined as the flows in which the density is negligible or can be a constant density (strict in-compressible). While, compressible is when the density variations within a flow are not negligible or fluid density varies significantly in response to a change in pressure.

A Newtonian fluid (named after Isaac Newton) is a fluid whose stress versus strain rate curve is linear and passes through the origin. For a Newtonian fluid, the viscosity depends only on temperature and pressure. Water and air are most commonly of Newtonian fluids. Non-Newtonian fluids are a type of fluid that is different from those of Newtonian fluids. It will be important in industries such as food, paint, plastic and pharmaceutical industries (Batchelor (2000)). Non-Newtonian fluid is a type of fluid whose flow properties differ in any way from those of Newtonian fluids in which shear stress is not directly proportional to deformation rate (Bejan (2004)).

There are several other possible approximations to fluid dynamic problem. The free convection problem is included in the field of buoyancy driven flow. Its often use the Boussinesq approximation because the density changes too small to be neglected except in the calculation on buoyancy forces where involves gravity.

In the fluid flow field, a numerical method such as Computational Fluid Dynamics (CFD) will be applied to solve the formulation of the problem. CFD is

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